

Nonlinear Dynamics and Turbulence

Edited by
G I Barenblatt
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Nonlinear Dynamics and Turbulence

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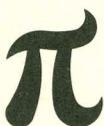
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